CLASSROOM ACTIVITY Stephen P. Cook

MAKING OBSERVATIONS (NEW MEXICO STYLE) TO ESTIMATE THE SIZE OF THE EARTH

Grade Level: Grades 7—12

Objectives—Students will appreciate:

1) that distances between two points on a map can be computed by measuring the distance between the points and multiplying by the map's scale
2) how moving on the surface of a sphere changes the direction one looks to see distant stars, and that moving greater distances produces proportionally bigger directional changes (in angles measured with respect to the horizon)
3) how measurements of the altitude of a bright star can be made from two different locations and these data used to estimate the Earth's size
4) how traveling in New Mexico to make such observations can give a sense of how big the Earth is and how one's eyes reveal this

Figure #1:When Canopus Culminates Above the Southern Horizon[1]
(be careful: all times shown are MST)

15-Oct	4 54 AM	15-Jan	10 53 PM
1-Nov	3 47 AM	1-Feb	9 46 PM
15-Nov	2 51 AM	15-Feb	8 50 PM
1-Dec	1 49 AM	1-Mar	7 55 PM
15-Dec	12 54 AM	15-Mar	7 00 PM
1-Jan	11 47 PM	1-Apr	5 56 PM

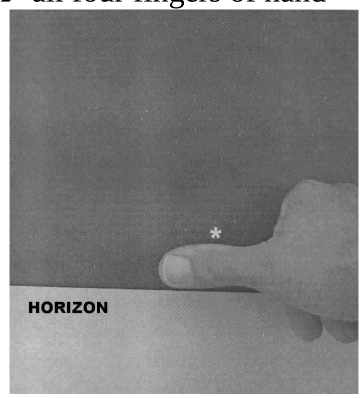
Teacher Background:

While travel and night-time observation enhance learning, this activity can be done entirely in the classroom. Students will gauge the Earth's circumference using the same method employed by Greek geographer Posidonius around 100 BCE. He measured the altitude of Canopus at its high point (culmination) at two different locations, then used simple math based on geometry.[2] Begin with a math review: of angles and their measurement, figuring distances by using a map with a scale, and about circles, including C =π D. If possible, facilitate students' climbing on a playground spherical jungle gym. Design this activity to help them understand the Figure #3 example (or related problem based on available playground equipment.) Describe this in terms of an ant seeing a star overhead (at the zenith) at one (RH) location, but seeing it 24 degrees away from overhead at another (LH) location. If the locations are separated by distance X (=3 feet in Figure #3), since 24 degrees is to 360 degrees as 1 is to 15, distance X and the circumference C of the sphere are in this same ratio. So C = 15 X (= 15 times 3 = 45 ft in the example of Figure #3).

Next, tell students they are about to replace the playground sphere with the Earth (assumed spherical) and use the horizon instead of the zenith for gauging directions. Define a star's altitude as the angle between the direction you look to see it and the horizon. Ask them what the altitude of a star seen overhead is? (Answer: 90 degrees) Ask how many degrees in a circle like the horizon? (Answer: 360 degrees)

Finally, tell your students about the star this activity is built around: Canopus[3]—in particular, three things about observing it: 1) it's best placed for viewing from New Mexico in late fall or winter, 2) you need to pick a spot with a clear view of the horizon, looking due south, and 3) you must know exactly when to look: within a few minutes of the times listed in Figure #1. Note from the latitude (35 degrees) of Albuquerque, the highest Canopus gets —at its culmination—is a mere 2 degrees in altitude above the horizon. Then, if you hold your thumb at arm's length, it should just fit between Canopus and the horizon as the middle part of Figure #2 shows. If you're farther south—say at the (32 degree) latitude of Las Cruces—you'll see this star a bit higher in the sky. How much higher? Challenge them to make these observations, compute the Albuquerque—Las Cruces distance based on a New Mexico map, and use all of this to get an estimate for the Earth's circumference!

Figure #2: Estimating Altitude Angles
Using parts of your hand at arm's length
1 deg ➔ width of your index fingernail
2 deg ➔ the fattest part of your thumb
5 deg ➔ all four fingers of hand



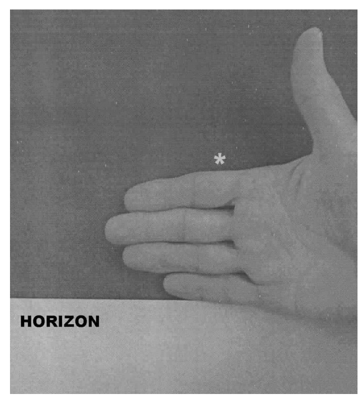
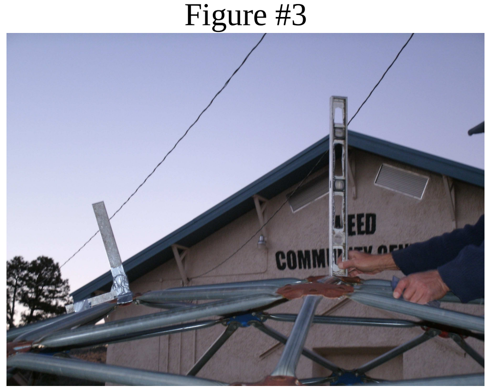
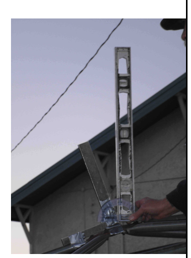


Figure #3



LH------ 3 ft ------RH



Method Overview: After review angles, map scales, an explanation of what it means to see a star at the zenith, on the horizon, or at some altitude—perhaps aided by playground exploration—you'll be working with three maps: #1 one of New Mexico, #2 one you'll create of Earth, and #3 the sky maps of Figure #4.[4]

Materials:
compass (to draw a circle), pencil, protractor, ruler, map of New Mexico w/ scale, tape measure, carpenter's level, builder's square, calculator

Method

MAP #1: On your New Mexico map, identify the scale. Establish the latitude and longitude of your location and mark it LOC1 on this map.[5] (Use 35.0 N, 106.7 W = -106.7 as default, near Albuquerque) For observation purposes, identify a second location in New Mexico widely separated in latitude from LOC1. (default: near Las Cruces 32.0 N, -106.7) Mark it LOC2. Locate where a point with LAT,LONG = 37.0 N, -106.7 is and mark it CH (just over the border in CO).

Exercises

1) Use a ruler and the scale to compute the distance between LOC1 and LOC2.

MAP #2: Draw a circle with diameter D = 8 inches by using a compass. Suppose this represents Earth, with diameter of roughly 8,000 miles. Label opposite points at top and bottom of this circle "N" and "S" to represent north and south poles and connect them with a line segment. Find a point on the circle half way between N and S and label it "E". Draw a line segment that passes through E and the circle's center and extends to the opposite side of the circle. Label it "equator" Recall latitude angles are measured north or south from the equator and use a protractor to mark locations on the circle at latitudes 37 degrees north and 53 degrees south. Label these as "CH" and "CZ" respectively. CZ is where Canopus culminates at the zenith; at CH it would then be seen on the horizon.

Exercises:

1) Figure out the scale of your drawing.
2) Draw in the directions to both the zenith and horizon from locations CH and CZ. Extend your lines pointing to the zenith back to the Earth's center so they intersect.
3) Moving on the circle from CH to CZ moves you through what angle (as figured from Earth's center) and what distance--both in miles and in terms of fraction of Earth's circumference?

MAP #2 and MAP #3—Exercises:

Using a 30 degree vertical extent for the sky map in Figure #4a and a ruler, figure out the scale in degrees/inch. Using this scale and a ruler, estimate the altitude of Canopus in each view for Figures #4a –4f and identify latitudes of the 6 observing sites. For your locations LOC1 and LOC2, repeat exercise 2) above on MAP #2, and after reviewing what you learned from Figures #3 & 4 and exercise 3) above, use your data to estimate the circumference of the Earth.[6,7]

Figure #4a

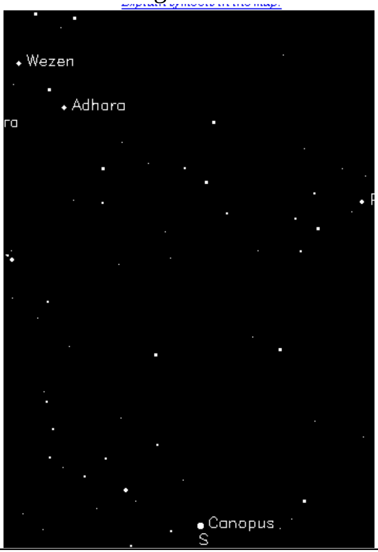


Figure #4b

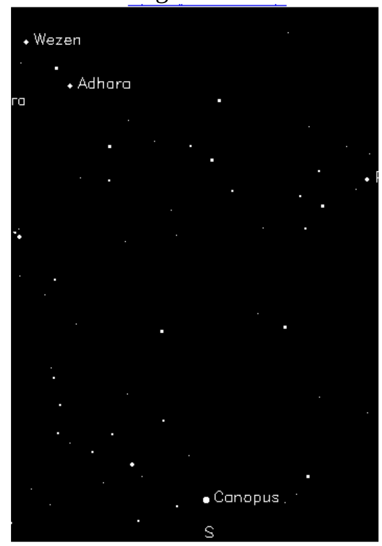


Figure #4c

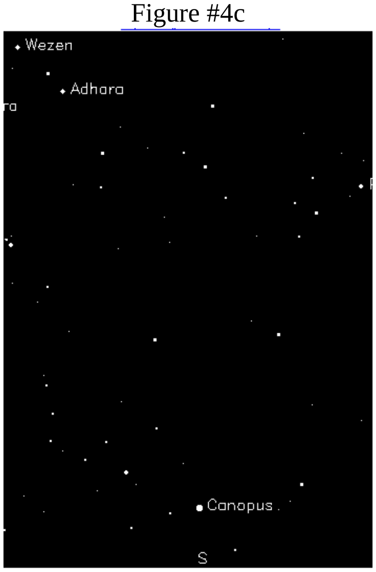


Figure #4d

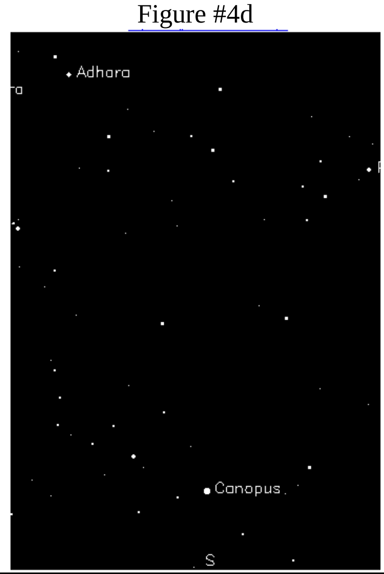


Figure #4e

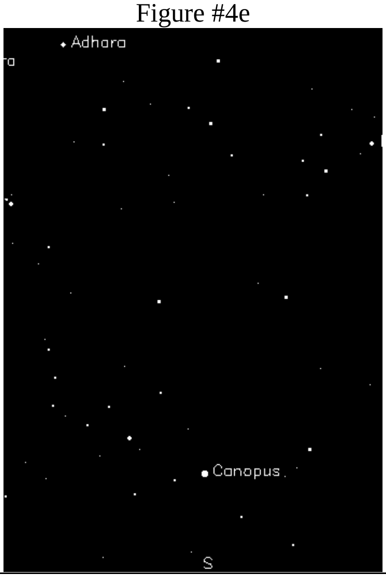
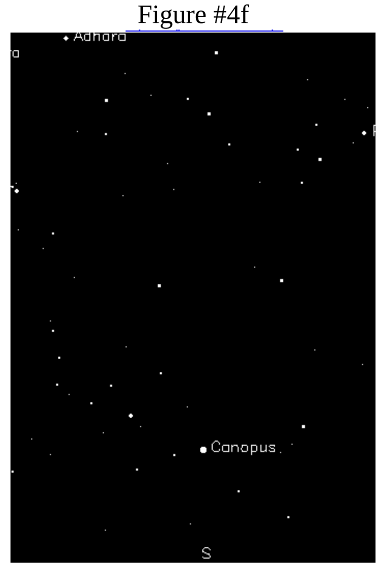


Figure #4f



Teacher Supplemental Notes:

1) Times in Figure #1 are for Albuquerque, they're good within one minute for Las Cruces. For anywhere else in New Mexico they'll be off by no more than ten minutes depending on location's longitude. For out of state, errors will depend on distance from the standard meridian of the time zone. For dates other than those listed, find the time by interpolating, using 4 minutes / day of change in time.

2) You may want to mention Erathostenes (276-195 BCE). For more background see http://en.wikipedia.org/wiki/History_of_geodesy

3) Canopus is the second brightest star in the sky, trailing only Sirius, but near the horizon extra atmosphere somewhat dims its light. Coincidentally, these two stars are in roughly the same part of sky—with Canopus being about 35 degrees farther south than Sirius.

4) The skymaps in Figure #4 were created using online software found at www.fourmilab/yoursky/horcustom.html

They were created for the following latitudes with corresponding values of Canopus' altitude (in degrees) at culmination:

Fig #4a Lat=36.0 max alt = 1	Fig #4b Lat=35.0 max alt = 2
Fig #4c Lat=34.0 max alt = 3	Fig #4d Lat=33.0 max alt = 4
Fig #4e Lat=32.0 max alt = 5	Fig #4f Lat=31.0 max alt = 6

5) the online Google Maps feature—brought up by right clicking on a location—"Where is this?" can be used to get LAT and LONG.

6) The distances between the two viewing locations —for Figure #2 Albuquerque and Las Cruces are roughly 200 miles apart—and the change of position—here 5 degrees minus 2 degrees =3 degrees—are directly proportional. So if you traveled thirty times that distance—200 miles x 30 = 6000 miles — the change of position would be 3 degrees x 30 = 90 degrees. This could be verified by noting at the New Mexico—Colorado border location CH Canopus would at best be essentially on the horizon with maximum altitude of 0 degrees, whereas 6000 miles to the south at location CZ in extreme South America it would be overhead at 90 degrees altitude. So the New Mexico/Canopus recipe for estimating the distance around the Earth would be to multiply the distance between its two largest cities by 30 x 4 = 120, or roughly 200 miles x 120 = 24,000 miles. You may want to ask students about the uncertainty in their result, and sources of error/limitations of the method (difficulty seeing horizon, assumes Earth is spherical, failure to account for atmospheric refraction, etc.)

7) Students who actually observe Canopus' culmination from two different locations could be asked to estimate its altitude—to nearest half of a degree—using their fingers/hand held at arm's length per Figure #2. Then, rather than matching their results to particular skymaps in Figure #4 and using the whole number latitudes and altitudes, more exact values could be used in their calculations.